# Vortical Flow Solutions Using a Time-Lagged, Thin-Layer, Navier-Stokes Algorithm

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## Abstract

A PROVEN inviscid algorithm (Program EAGLE - Flow Solver)[1-3] has been modified to use the flux-difference split scheme of Roe[4] to solve the time-lagged, thin-layer approximation to the Navier-Stokes equations.[5,6] Calculations are presented evaluating the ability of the time-lagged, thin-layer Navier-Stokes algorithm to obtain transonic solutions for lifting bodies at incidence angles sufficient for the flow to be dominated by large-scale free vortices in the leeside flowfield.

## Nomenclature

| | |
|---|---|
| $A^+$, $B^+$, $C^+$ | = Jacobians of the split flux vectors (positive eigenvalues) |
| $A^-$, $B^-$, $C^-$ | = Jacobians of the split flux vectors (negative eigenvalues) |
| $F$, $G$, $H$ | = curvilinear coordinate flux vectors |
| $Q$ | = dependent variable vector |
| $R$ | = residual vector |
| $U$, $V$, $W$ | = contravariant velocities |
| $X$ | = intermediate solution vector |
| $\tau$ | = transformed time variable |
| $\phi$ | = circumferential orientation angle (deg) |

## Contents

A three-dimensional, multiblock, time-dependent, thin-layer Navier-Stokes (TLNS) solution algorithm has been constructed using finite-volume discretization. An implicit approximate factorization of the Euler equations is a two-pass algorithm[1-3] given by

$$[I + \Delta\tau(\delta^i_\xi A^+_\cdot + \delta^i_\eta B^+_\cdot + \delta^i_\zeta C^+_\cdot)]X^1 = -\Delta\tau \boldsymbol{R}^n \tag{1}$$

$$[I + \Delta\tau(\delta^i_\xi A^-_\cdot + \delta^i_\eta B^-_\cdot + \delta^i_\zeta C^-_\cdot)]\Delta Q^n = X^1 \tag{2}$$

where the residual vector, $\boldsymbol{R}^n$ is defined as

$$\boldsymbol{R}^n = [\delta^e_\xi F + \delta^e_\eta G + \delta^e_\zeta H]^n \tag{3}$$

This two-factor scheme has been shown to be attractive in that it involves the solution of a sparse block lower triangular system of equations using a simple forward substitution followed by the solution of a sparse block upper triangular system using a backward substitution. The treatment of the implicit and explicit operators are patterned after the recommended approach of Liou and van Leer.[7] The implicit operator uses the flux-vector splitting technique of Steger and Warming.[8] For the explicit operator, the flux-difference split scheme of Roe[4] is employed, based on the Navier-Stokes calculations of Whitfield et al.[2]

Viscous solutions are obtained by employing the efficient two-pass algorithm of Eqs. (1) and (2) and explicitly adding the contribution of the diffusive terms to the residual vector in Eq. (3), thereby time lagging the viscous effects, yielding the modified residual vector

$$\boldsymbol{R}^n = [\delta^e_\xi F + \delta^e_\eta (G - G^d) + \delta^e_\zeta H]^n \tag{4}$$

This treatment of the diffusive terms reduces the cost of normal viscous calculations substantially by saving the cost associated with computing and storing the viscous flux Jacobians. Closure of the governing equations is obtained using the algebraic turbulence model proposed by Baldwin and Lomax.[9]

The configuration employed in this analysis is a generic elliptic missile shape with a cross-sectional ellipticity ratio of 3:1. The body has an axial power-law distribution with exponent of 0.5.[10] The multiblock grid generated is a three-block algebraic grid about a 180 deg section of the elliptic cross section. The blocks are separated axially and yield a total grid density of 100 × 65 × 65. Grid lines are concentrated in the boundary layer to a $y^+$ value of approximately 1.0 and in the transverse direction in the general region of the expected vortical flow pattern ($\phi$ = 65 deg).

Calculations have been completed for varying Mach numbers from 0.95 through 1.20, angles of incidence between 12.8 and 13.0 deg and Reynolds numbers of approximately 3 × $10^6$ per foot. Turbulent flow is assumed to be fully developed along the entire length of the configuration. All solutions converged to at least three orders of magnitude for the L2 Norm and two and a half orders for the maximum residual in both density and energy. Figure 1 shows the circumferential pressure distribution for Mach 1.05 at 12.9 deg at a 53% axial location ($X/L$). The experimental data[10] shows the expansion of the flow as it accelerates from the windward to leeward side of the body, as well as the minimum pressure pattern indicative of vortical flow at the 65-deg circumferential position ($\phi$). The viscous solution (TLNS) appears to overexpand at the outboard meridian (90 deg circumferential location) and to underpredict the magnitude of the leeside rollup of the vortex sheet. Constant vorticity contours for this axial location are shown in Fig. 2 in which the rollup of both the primary and secondary vortices are in evidence.

Figure 3 shows the circumferential pressure distribution for the Mach 1.20 case at 13.0 deg at an axial location of 98%

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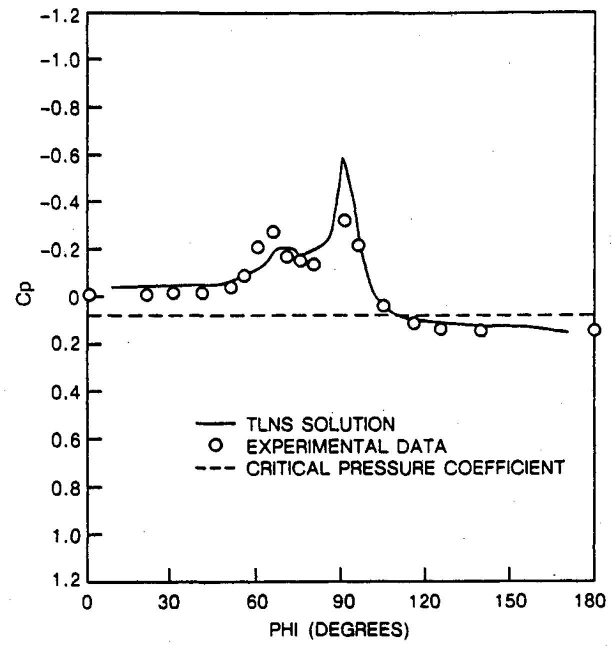


**Fig. 1 Circumferential pressure distribution: Mach = 1.05, Alpha = 12.9 deg, Axial = 53%.**

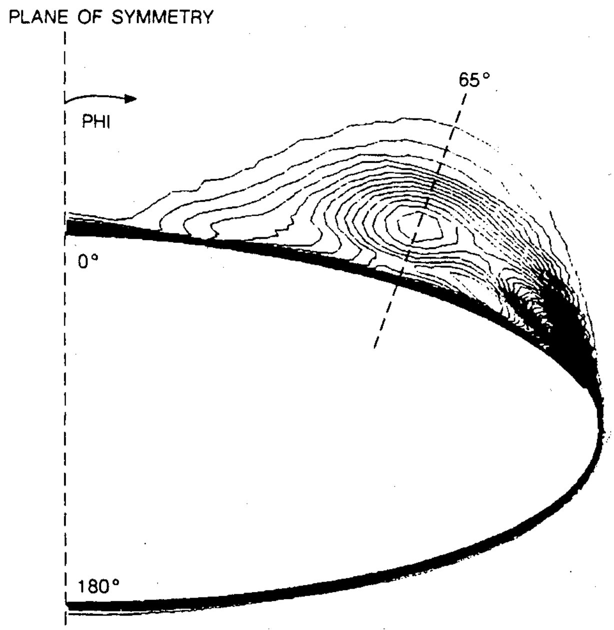


**Fig. 2 Constant vorticity contours: Mach = 1.05, Alpha = 12.9 deg, Axial = 53%.**

($X/L$). The numerical solution (TLNS) agrees well with experimental data in predicting the transverse expansion of the flow and the reduced magnitude of the minimum pressure pattern at the 65-deg circumferential position $\phi$, indicating the migration of the vortex off the body surface.

The minimum pressure pattern, indicative of the presence of vortical flow and experimentally evident for all transonic cases run, was adequately resolved for solutions above sonic conditions. The migration of the primary vortex off the configuration surface was well predicted although slightly misplaced in a few locations when compared to experimental data. This was not the case for transonic runs under sonic conditions. It appears likely that the well-documented failings of the Baldwin-Lomax model in resolving large free-scale vortices has corrupted the TLNS solution. Inherent to the turbulence model is the basic problem of improperly determining the length and velocity scales in the vortical flowfield. This results in too

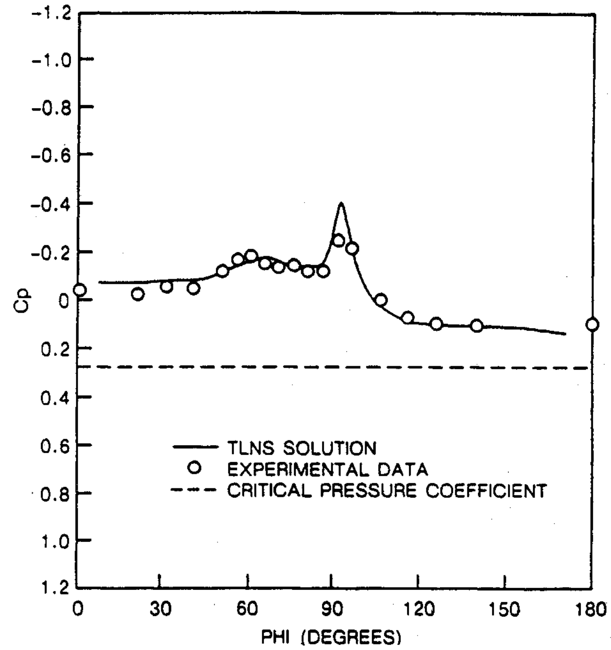


**Fig. 3 Circumferential pressure distribution: Mach = 1.20, Alpha = 13.0 deg, Axial = 98%.**

large an outer viscosity increment that serves to diffuse the vortical pattern, thereby delaying the initial rollup of the primary vortex and reducing the impingement on the body surface.

The time-lagging approach using the thin-layer Navier-Stokes approximation appears sound for obtaining solutions to transonic vortical flow over generic lifting body configurations. In general, viscous solutions compared favorably with available experimental data and any shortcomings appear to be caused by the closure model employed.

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